Study of Light Scattering by Small Particles

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7/1/2003
Light Scattering Study

- Study of light scattering by small PSL spheres using NIST SCATMECH C++ Library code, and comparing the results with DDSURF and Xtreme.

- Various sizes of PSL spheres on smooth Silicon substrate illuminated with three wavelengths (532, 350, and 266nm), P and S polarization, and various incident angles. For comparison purposes the NA was set to 0.41, the value of our current high mag lens. Results indicate all 3 sources of data have good agreement.

- Assuming identical power density, it is inferred that we should be able to detect 55nm and 42nm PSL spheres @ 350nm (P) and @ 266nm (P) respectively, based on current capability of 80nm PSL detection @ 532nm (P).

- For small particles (relative to the wavelength), P-polarization produces higher scattered intensity than S-polarization does. However, the top mounted CCD is not preferred for collecting P-polarized scattered light.

- $n$ and $k$ values used for PSL and Silicon:

<table>
<thead>
<tr>
<th>Lamda nm</th>
<th>Si n</th>
<th>Si k</th>
<th>PSL n</th>
<th>PSL k</th>
</tr>
</thead>
<tbody>
<tr>
<td>266</td>
<td>1.850</td>
<td>4.432</td>
<td>1.670</td>
<td>0</td>
</tr>
<tr>
<td>350</td>
<td>5.475</td>
<td>3.002</td>
<td>1.640</td>
<td>0</td>
</tr>
<tr>
<td>532</td>
<td>4.152</td>
<td>0.052</td>
<td>1.598</td>
<td>0</td>
</tr>
</tbody>
</table>
Model Comparison
Model Comparison Summary

- Compared model predicted data generated from DDSURF and NIST-SCATMECH with actual data from Xtreme, an optical dark-field wafer inspection tool manufactured by INSPEX.
- All 3 data sets (both P and S polarization) have good agreement.
- The SCATMECH predicted less scattering than what the DDSSURF predicted:
  - 37.5% less for P-pol.
  - 31.7% less for S-pol.
- Xtreme data exhibited some variations, but the trend lines showed good agreement with data from both models.
  - Xtreme data were sub-pixel interpolated by a factor of 10 in both X and Y directions.
  - The interpolated pixels above a threshold (20 for P and 25 for S) were integrated and normalized for comparison purposes.
NIST assumes incident from $-X$ but DDSURF from $+X$. The scattering and the azimuth directions for both models use the same convention, $+X$. 
DSC Comparison

- Hemisphere plots for DDSURF DSC (left) and SCATMECH DSC (right) under identical conditions. Note that the coordinate systems are 180° apart along the X axis.
DSC Comparison

- Differential Scattering Cross-section (DSC) maps by DDSURF (left) and SCATMECH (right), obtained under identical conditions. The map from DDSURF was rotated 180 degrees in the X direction for comparison purposes.
- Both models suggest stronger back-scattering (strongest @ $\theta_s = 63\pm1^\circ$)

Cscat = $7.9759e-005$  \quad Cscat = $5.7472e-005$
Based on DSC difference and ratio between the two models, it can be found that:

- The DDSURF predicted DSC is larger than the SCATMECH's prediction at every point, with an average ratio of 1.37.
- The largest discrepancy lies in the forward direction (0 degree).

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.9207e-006</td>
</tr>
<tr>
<td>Std</td>
<td>3.3752e-007</td>
</tr>
<tr>
<td>Min</td>
<td>3.1953e-009</td>
</tr>
<tr>
<td>Max</td>
<td>6.7741e-006</td>
</tr>
</tbody>
</table>

Mean ratio = 1.3671
Response to Various PSL Sizes

Total Scattered Cross-Section Comparisons
(532nm P & S, 50 deg Incident, NA = 1)
Comparison with Xtreme Data (P)

Scattered Cross-Section Comparison
(532P, 80nm PSL, NA = 0.4)
Comparison with Xtreme Data (S)

Scattered Cross-Section Comparison
(532S, 80nm PSL, NA = 0.4)
Xtreme Data (80nm PSL, P-pol, max power)
Xtreme Data (80nm PSL, S-pol, max power)

\[ \theta = 50^\circ \]

\[ \theta = 55^\circ \]

\[ \theta = 60^\circ \]

\[ \theta = 65^\circ \]

\[ \theta = 70^\circ \]

\[ \theta = 75^\circ \]

\[ \theta = 80^\circ \]
NIST SCATMECH
Results
NI ST SCATMECH Scattering Model

The SCATMECH library, which was developed by the National Institute of Standards and Technology (NIST), consists of a set of C++ object classes which allow programmers to manipulate polarimetric quantities. It adds to standard C code data types corresponding to polarimetric quantities and optical properties of materials. It further adds a base class which acts as a socket for light-scattering models, and includes a variety of specific models, e.g., particles, subsurface defects, and microroughness.

Surface light scattering from a small particle was simulated with the Rayleigh particle Bidirectional Reflectance Distribution Function (BRDF) model:

$$BRDF = \frac{16\pi^4}{\lambda^4} \left( \frac{n_p^2 + 1}{n_p^2 + 2} \right)^2 \frac{d^6}{\cos \theta_s \cos \theta_i} \times |q_{ij} \cdot \vec{e}|^2$$

Schematic of BRDF model showing the ellipsometry of intensity relative to both the p and s polarization axes.
NIST SCATMECH BRDF Model

- NIST SCATMECH DSC computation
  - The bidirectional reflectance distribution function (BRDF) is the fractional power scattered per projected solid angle.
  - The Differential Scattering Cross-section (DSC) and the total integrated intensity can be computed using BRDF as follow:

\[
BRDF = \frac{dI_s}{I_i (d\Omega \cdot \cos \theta_s)}
\]

\[
DSC = \frac{BRDF (\cos \theta_i \cdot \cos \theta_s)}{\rho}
\]

\[
\frac{I_s}{I_i} = \sum_{\phi_s=0}^{\pi/2} \sum_{\theta_s=0}^{\pi/2} BRDF \cdot \cos \theta_s \cdot \sin \theta_s \cdot \Delta \theta_s \Delta \phi_s
\]
The NIST SCATMECH scattering model has been confirmed experimentally with their MHPOSI, and the results have been presented in several publications.
PSL Scattered Cross-Section for 532nm
P&S, NA = 0.41 (Size vs. Incident Angles)
PSL Scattered Cross-Section Comparison
(3 wavelengths, P-polarization, NA = 0.41)
PSL Scattered Cross-Section Comparison (3 wavelengths, S-polarization, NA = 0.41)

NA=0.41 Scattered Cross-Section from 3 PSL Spheres (40, 60, and 80nm)
PSL Scattered Cross-Section Comparison (P-polarization Ratio, NA = 1)

PSL Scattered Cross-Section Ratios

Average gain (50° -- 80°) = 23.1

Average gain (50° -- 80°) = 7.8
Scattered Intensity Comparison
($\theta_\text{i} = 50^\circ$, NA = 0.41, unit power density)

![Graph showing scattered intensity comparison with different wavelengths and diameters.](image-url)
40nm PSL
Scattered CS from a 40nm PSL with a 266nm Laser (various incident angles)

266nm Laser on 40nm PSL (NA = 0.41)

Collection Efficiency (NA = 0.41)
40nm PSL
Scattered CS from a 40nm PSL with a 350nm Laser (various incident angles)

350nm Laser on 40nm PSL (NA = 0.41)

Collection Efficiency (NA = 0.41)
40nm PSL
Scattered CS from a 40nm PSL with a 532nm Laser (various incident angles)

532nm Laser on 40nm PSL (NA = 0.41)

Collection Efficiency (NA = 0.41) 40nm PSL
PSL Scattered Intensity Comparison (40nm PSL, P-pol, 50° incident hemiplots)
PSL Scattered Intensity Comparison (40nm PSL, P-pol, 80° incident hemiplots)
PSL Scattered Intensity Comparison (40nm PSL, S-pol, 50° incident hemiplots)

266S

350S

532S
PSL Scattered Intensity Comparison (40nm PSL, S-pol, 80° incident hemiplots)
Scattered CS from a 60nm PSL with a 266nm Laser (various incident angles)

![Graph 1: 266nm Laser on 60nm PSL (NA = 0.41)]

![Graph 2: Collection Efficiency (NA = 0.41) 60nm PSL]

- **Scattered cross-section (um^2)**
  - 266P
  - 266S

- **Ratio (NA = 0.41 / Total)**
  - 266P
  - 266S
Scattered CS from a 60nm PSL with a 350nm Laser (various incident angles)

350nm Laser on 60nm PSL (NA = 0.41)

Collection Efficiency (NA = 0.41)
60nm PSL
Scattered CS from a 60nm PSL with a 532nm Laser (various incident angles)

532nm Laser on 60nm PSL (NA = 0.41)

Collection Efficiency (NA = 0.41)
60nm PSL
PSL Scattered Intensity Comparison
(60nm PSL, P-pol, 50° incident hemiplots)
PSL Scattered Intensity Comparison (60nm PSL, P-pol, 80° incident hemiplots)
PSL Scattered Intensity Comparison (60nm PSL, S-pol, 50° incident hemiplots)
PSL Scattered Intensity Comparison (60nm PSL, S-pol, 80° incident hemiplots)
80nm PSL
Scattered CS from a 80nm PSL with a 266nm Laser (various incident angles)

266nm Laser on 80nm PSL (NA = 0.41)

Collection Efficiency (NA = 0.41)
80nm PSL
Scattered CS from a 80nm PSL with a 350nm Laser (various incident angles)

![Graph 1: Scattered cross-section (nm²) vs. Incident angle (deg)]

![Graph 2: Collection Efficiency (NA = 0.41) 80nm PSL vs. Incident angle (deg)]
Scattered CS from a 80nm PSL with a 532nm Laser (various incident angles)
PSL Scattered Intensity Comparison (80nm PSL, P-pol, 50° incident hemiplots)
PSL Scattered Intensity Comparison (80nm PSL, P-pol, 80° incident hemiplots)
PSL Scattered Intensity Comparison (80nm PSL, S-pol, 50° incident hemiplots)
PSL Scattered Intensity Comparison (80nm PSL, S-pol, 80° incident hemiplots)
Effect of Incident Angles (266nm P on 40nm PSL)

\[ \theta = 0^\circ \]
\[ \theta = 10^\circ \]
\[ \theta = 20^\circ \]
\[ \theta = 30^\circ \]
\[ \theta = 40^\circ \]
\[ \theta = 50^\circ \]
\[ \theta = 60^\circ \]
\[ \theta = 70^\circ \]
\[ \theta = 80^\circ \]
\[ \theta = 85^\circ \]
\[ \theta = 89^\circ \]