PROBABILISTIC FATIGUE DAMAGE MODELING FOR THE RELIABILITY BASED ROTORCRAFT COMPONENT LIFE ASSESSMENT USING HUMS

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ABSTRACT

A probabilistic fatigue damage modeling methodology was investigated to address the current U.S. Army's six-nine reliability requirement in the safe-life approach, anticipating a wider use of HUMS (Health Usage Monitoring System) usage data for more accurate rotorcraft component life assessment and a possible UBM (Usage Based Maintenance) credit. The probability distributions of the two key parameters in the fatigue life assessment, i.e. the loads and the material strength, were suitably modeled to ensure six-nine reliability for situations when the HUMS data would provide accurate account of operational usages. The reliability based fatigue life assessment using HUMS usage data was carried out in two separate parts: 1) the probabilistic modeling of fatigue damages, and 2) assessment of the effect of HUMS usage data in the safe life of a component. This paper concerns the first part of the two, the probabilistic modeling of fatigue damages.

Both the constant standard deviation (CSV) approach and the constant coefficient of variation (CCV) approach, commonly used in the industry to reduce the fatigue strength in constructing working S-N curve, were modeled. The probability distributions of each flight regime load and the material strength were modeled using Weibull probability distribution functions. The study utilized the composite worst case (CWC) usage spectrum to compare the life with the current legacy life which was calculated based on a deterministic lifing methodology. Trends of component fatigue lives with varying degrees of reliability were also investigated and presented in this paper.

INTRODUCTION

The analysis procedures for fatigue damage assessment described in this study focus on the rotorcraft component life assessment using HUMS data, which enables one to update the operational usage spectrum of individual aircraft or a fleet of aircraft, fulfilling goals of the condition based maintenance (CBM) to reduce burdensome maintenance tasks, increase aircraft availability, improve flight safety and reduce maintenance cost [1, 2].

However, it has been one of the major issues and research topics whether the use of accurate operational usage data collected by HUMS for the component life assessment would possibly decrease the reliability of component life by eliminating the conservatism built in the design usage.

It has been generally accepted in the rotorcraft community that the conventional fatigue life assessment methodology of the safe-life approach, so called the "Legacy Method" approximately provides a .999999 (six nines) reliability level through deterministic calculation of key parameters with large conservations. The six-nine reliability in the Legacy method is explained as three-nines from strength, two-nines from loads, and one-nine from usage, and generally taken from the “3-sigma” fatigue working curve margin, the "high envelope" treatment of flight loads, and the “worst case” assumption on usage, respectively [3, 4]. This six-nines reliability requirement (or a probability of failure of one in a million) on the component retirement time (CRT) for the safety critical rotorcraft dynamic component was established by the U.S. Army through its Light Helicopter program in the late 1980’s [5, 6], and has been generally accepted as a standard reliability requirement in the rotorcraft community [7-11].

The assessment of actual reliability of the safe-life fatigue methodology up to a level of six-nine (.999999) accuracy is an extremely challenging task due to the involvement of multiple statistical parameters such as the material strength, many flight conditions and usages, and the cumulative nature of fatigue damage. To resolve this reliability issue among others, there were two round robin studies organized by the US Army/AHS, and so far, no one methodology universally accepted by the community has been established [6-11].

This reliability issue on using HUMS for component fatigue life assessment has been one of the major obstacles for wider use of HUMS in the field, and was studied as a part of supporting the FAA Advisory Circular Circular AC 29-2C MG 15, “Airworthiness Approval of
Rotorcraft Health Usage Monitoring Systems (HUMS)” [1]. Considering its complexity, it was tackled in two phases by separating out the probability modeling of the fatigue methodology accounting for variability in material strength and flight loads from the reliability issue arising by the introduction of HUMS usage data in the fatigue life assessment process. The Part I of the study, “Probability Modeling of Rotorcraft Fatigue Damage”, is described in this paper, and the Part 2, “Reliability Based Fatigue Life Assessment Using HUMS”, will be described in a separate paper.

PROBABILITY MODELING OF FATIGUE DAMAGE

Since the deterministic approach of current structural safe-life assessment methodology has a limitation to accurately quantify the reliability of assessed component retirement time, a probabilistic approach is studied to meet the high level of reliability requirement on retirement time of rotorcraft dynamic components. For this effort, the probabilistic nature of fatigue damage is briefly reviewed, followed by the proposed probabilistic modeling of it in a two phase approach: modeling fatigue strength and flight loads first, followed by the use of HUMS usage data.

Probabilistic Nature of Fatigue Damage

As the first S-N diagram was conceived by August Wöhler in 1870, the structural fatigue damage phenomena, particularly for rotorcraft dynamic components, are considered as probabilistic by nature. The three major elements of structural fatigue damage, the material fatigue strength, the loads applied and the cycles or the usage of operations. Each of these elements is subject to large variability: scatter in test data of the fatigue material strength, severe variability of loads in multiple flight conditions and different operational usages on each component installed on individual aircraft.

Since these three major elements are subject of probabilistic distributions, the structural fatigue damage is a phenomenon of joint probability, as described in [6]. Substantial research efforts were conducted and numerous technical papers published since the 1950’s to address the variability of fatigue parameters and proposed alternative methodologies for better way to predict structural life than the current deterministic safe-life methodology, culminating in the two round robin studies mentioned earlier [6-8].

Deterministic Models

The conventional safe-life fatigue assessment methodology employs deterministic models for the key parameters with intentionally imposed conservatism in them. Eq.1 is a typical formula based on the Weibull failure rate equation to represent the mean fatigue strength of structural components:

\[
\frac{S}{E_{\text{inf}}} = 1 + \frac{\beta}{N^T}
\]

Eq. 1

where \(S\) is the stress (or load), \(E_{\text{inf}}\) is the endurance limit at infinity, \(N\) is the allowable number of cycles, and the two parameters \(\beta\) and \(\gamma\) define the scale and shape of the curve. In the Palmgren-Minor (PM) linear damage accumulation model, the measure of damage is simply the cycle ratio with the basic assumptions of constant work absorption per cycle from the constant amplitude vibratory load, and the characteristic amount of work absorbed at failure:

\[
d = n/N
\]

Eq. 2

where \(n\) is the number of cycles applied. The energy accumulation, therefore, leads to a linear summation of cycle ratios of damages for various loads applied:

\[
D = \sum \frac{n_i}{N_i}
\]

where the subscript \(i\) denotes the \(i\)-th segment of applied loads. The structure is deemed to fail when the accumulated damage \(D\) becomes 1.

As described earlier in the Introduction, the conventional safe-life approach, Legacy method, employs deterministic values of the key parameters with intentionally imposed conservatism: a “3-sigma” fatigue working curve margin, a “high envelope” treatment of flight loads, and a “worst case” assumption on usage. This approach approximately provides the six-nine reliability level [12].

It is worthwhile to note that the worst case usage assumption in the conventional approach provides approximately one-nine of the six-nine reliability goal for the “average” usage, which is a simple conservative reduction of the worst case usage. That means that if the design usage spectrum is not maintained in average sense, the six-nines reliability is no longer valid, or guaranteed [6, 12].

Probabilistic Models

The reliability of structural fatigue damage is very complex as described in the introduction, and to quantify the actual reliability of measured CRT up to the level of six-nine accuracy by the simple and convenient deterministic model of fatigue damage described above has limitations. A significant effort has been taken since the first round robin in 1987 to develop an accurate and comprehensive probabilistic methodology which can be universally acceptable as a
standard practice. Several promising proposals were made earlier, but no one method has been established as an acceptable standard reliability based fatigue life assessment methodology [6-13]. Through this renewed effort, a comprehensive probabilistic approach based on the joint probability concept proposed during the first round robin study was studied with new interpretations and implementation of the recommendations made from the two round robin studies and as applicable to the use of HUMS for its usage based maintenance (UBM) credits.

The proposed approach takes advantage of HUMS usage which simplifies the complex reliability problem into a joint probability modeling of two key parameters, material strength and flight loads, with the assumption that the HUMS data will provide an accurate operational usage without variability. The variability in HUMS usage is expected between aircraft, which is to be taken into account during the assessment of fleet component retirement times, but it is assumed there will be no usage variability from HUMS for individual aircraft other than possible minor machine errors or other discrepancies which can be handled by proper mitigation strategies.

The separation of the usage part enables the probability modeling to focus on an accurate joint probability modeling of material strength and multiple flight load conditions. Figure 1 shows an example of this joint probability of load and strength at the 1.0 million allowable cycles. The overlapped area of the two probability density function (pdf) curves represents the joint probability of the damage. Here, we can safely assume that the load variable and the strength variable are independent, and that the damage \( D_r \) incurred by each regime \( r \) is represented by the integral of the joint probability of the two pdf’s, represented by the convolution \( f \ast g \), multiplied by the damage function in Eq. 2.

\[
D_r(n_r) = \int_0^\infty \int_0^\infty n_r \frac{n}{N} (f \ast g) d\delta ds \quad \text{Eq. 4}
\]

In Eq. 4, the \( \delta \) and \( s \) are the increments of loads and strength, respectively.

The total damage from all flight regimes is the algebraic summation of the damage from each regime, as shown in Eq. 3. As can be seen in Eq.4, it is also assumed that only the vibratory load of each regime is the concern and it does not become negative, which is a typical assumption used in the retirement life assessment of rotorcraft dynamic components.

Modeling of Fatigue Strength Distribution

A typical structural fatigue test results in a substantial data scatter. W. Weibull addressed the scatter of fatigue strength in the early 1950’s and described it as the P-Case A and P-Case B [13]. Figure 2 shows example distributions of structural fatigue test data where the probability density curves of the fatigue strength in vertical and horizontal directions are schematically drawn for the strength distribution (P-Case A) and the life distribution (P-Case B), respectively [14]. A proper stochastic approach needs to model both probability distributions in strength and time.

The material fatigue strength is typically represented by the formula shown in Eq.1 using the Weibull failure rate function. In the conventional safe-life approach, there are typically two methods to impose conservatism in the material strength by reducing the mean strength, vertical changes in Figure 2. One
method uses the constant coefficient of variation (COV), and the other uses the constant standard deviation (STD), as we will refer them as CCV and CSD, respectively. Rotorcraft manufacturers use both methods based on their extensive experiences and accumulated fatigue test databases, and have been successfully estimating CRT’s by judiciously utilizing values of COV or STD.

However, as demonstrated in the first round robin study [6], the use of different methods of CCV or CSD could become a major source of errors in CRT estimation, and, as a result, it was called to establish a uniform methodology for developing working S-N curve. Both approaches have pros and cons, and currently, it is not clear which method is better than the other method, and the choice is left to the manufacturers. Hence, both strength reduction methods of CCV and CSD were implemented in this probability modeling as an option. But, since all available Legacy CRT examples were assessed using the CCV method, only the CRT’s based on the CCV method are shown here to compare results on the same basis.

There is a need to implement another reduction of the mean strength curve, called life reduction, horizontal changes in Figure 2. This life reduction scheme is to address the slow-down of the S-N curve slope observed from typical metals in the low-cycle region due to the plasticity effect of high loads, and implemented by some manufacturers. In Figure 3, the working S-N curve in red color is composed by using the CSD method with the typical life reduction factor of 5. The working curve in blue is the one composed by using the CCV method with the same life reduction factor of 5. Note that the life reduction portion of the blue curve is hidden behind the red curve on the far left side due to the use of the same reduction factor. It should also be noted that the CSD method is a simple downward shift of the mean S-N curve in black, while the CCV method proportionally reduce the mean S-N curve, more reduction on the low-cycle side, that results in the shape change of the curve.

Eq. 5 represents the working S-N curve obtained by modifying the formula in Eq. 1 where a is the CCV strength reduction factor, b is the CSD strength reduction factor, and c is the life reduction factor. Either a or b is to be used depending on the selection of the CCV method or the CSD method, respectively.

\[
\frac{(1-b)S}{aE_{inf}} = 1 + \frac{\beta}{(cN)^y}
\]

Eq. 5

Figure 3. Comparison of Working S-N Curves

Typically, a 3\(\sigma\) (three standard deviations) is used for the strength reduction. As pointed out in [6], different strength reduction methods could result in a big difference in the estimated CRT’s. As an example, the estimated retirement time of the Main Rotor (MR) Rotating Swashplate for the helicopter model used in this study using the CCV method was 10,011 hours, while the CSD method estimated 19,782 hours for the same component, almost doubled the life of the CCV method.

All of these variations in working S-N curve construction make the probability modeling much more complicated, but is implemented in the proposed methodology as options to accurately compare the result with the current Legacy method for the purpose of validation and verification (V&V) of the developed model.

The dashed black curve in Figure 4 represents the mean S-N curve with a life reduction factor of 5, while the blue curve represents the working S-N curve from the CCV method and the dashed blue curve represents the life reduction portion of the working curve at lower cycles to failure. The vertical distribution of the endurance limit at 10^6 cycles is shown, overlaid at the lower right corner in Figure 4, and following the mean S-N curve representing the scatter of the material strength, according to the choice of the CCV or CSD method.

As shown in Figure 5, the vertical distribution of the material strength scatter will be constant when the CSD method is used, and will become proportionally wider when the number of cycles becomes smaller for the CCV method.
This strength distribution can be modeled by using a typical three-parameter Weibull pdf shown in Eq. 6.

\[ g(x; \lambda, \kappa, \gamma) = \frac{\kappa}{\lambda} \left( \frac{x-\gamma}{\lambda} \right)^{\kappa-1} e^{-\left(\frac{x-\gamma}{\lambda}\right)^{\kappa}} \]  

Eq. 6

Figure 6 shows an example of a 3-dimentional view with isolines of probability densities for the modeled strength distribution for the same Main Rotor (MR) Rotating Swashplate in Figures 3 and 4, where the CCV strength reduction method with the life reduction factor of 5 was used. The slope change due to the life reduction is visible on the upper left corner.

The above fatigue strength distribution model in the allowable cycle - load/strength space was then transformed into the load-strength joint probability space, which gave us a better modeling and more accurate numerical integration.
Modeling of Flight Load Distributions

There is a substantial variation in the magnitude of measured loads tested under a given flight condition. This can be caused by a number of things, including even the pilot skill, as shown in Figure 7. In the deterministic approach, a conservative value selected by the “top-of-the-scatter” (TOS) or high-envelope approach, typically selecting a value at 95% or 2\(\sigma\), is used.

Modeling of this variability of load in each flight condition, or regime, is a rather simple process when sufficient flight load data are available. Fortunately, there is such a case where extensive flight test data has been collected, and the same mid-size utility helicopter type also has accumulated substantial amount of HUMS usage data.

The flight test data collected for several hundred different flight conditions were grouped down to 57 regimes for which their fatigue damage rates per second or per occurrence were defined through fatigue substantiation tests. Multiple flight load data points for same flight regimes were collected during the flight test program, and the number of data points varied from a few points to several hundred points. Figure 8 shows the distribution of these data points for various regimes.

Figure 7. Example Distribution of Regime Max. Vibratory Loads

Note that each data point in the plot represents the maximum vibratory load within the regime cycles which is normally used in the conventional safe-life method to be conservative. These data points were then used to generate a two-parameter Weibull pdf, for each regime.

Figure 8 shows the frequency of data points with its Weibull pdf curve for the 2.0 Pull-out flight maneuver, as an example.

Weibull pdf curves representing the push rod load distribution for several selected regimes are shown in Figure 9. A large variation can be seen in the plot, from a very narrow scatter, representing Taxing or Hover, to a much wider scatter of loads, such as for Pullouts.

Modeling of Load-Strength Joint Probability

Weibull pdf curves represent the push rod load distribution for several selected regimes are shown in Figure 9. A large variation can be seen in the plot, from a very narrow scatter, representing Taxing or Hover, to a much wider scatter of loads, such as for Pullouts.

Once probabilistic models for the strength distribution and load distributions become available, the fatigue damage could be calculated by introducing usage, the
third element of the component to fatigue damage. To validate the proposed joint probability model, the same CWC block spectrum used in the Legacy method was used for comparison purposes. This CWC block spectrum is described later.

Having the probability models of load and strength as well as the CWC usage spectra available, the fatigue damage can be calculated using the damage formula shown in Eq. 4. As can be seen in the formula, the joint probability and hence the fatigue damage are defined in the physical space of allowable cycles and strength (or load). It was discovered that this approach was more sensitive to the increment size of numerical integration and could result in more damage due to the increased uncertainty on the flat side of the S-N curve, or the high cycle region, where it is more difficult to model both the vertical and horizontal probability distributions of strength. It was observed that flatter S-N curves, such as that used for Pitch Control Horn when compared to the one used for MR Swashplate, resulted in more errors in damage calculation.

Transforming the formula defined in the physical space into the load-strength joint probability space, as suggested in the first round robin study [6], provided a more accurate and efficient numerical integration. The regime damage formula of Eq. 4 was transformed to the formula shown in Eq. 7 where the \( f_r(x) \) with a tilde on \( f \) and \( g \) in Eq. 6 is the convolution of normalized pdf’s of each regime load and strength in the probability space.

\[
D = \sum_r \int_0^1 \int_0^1 \frac{n_r}{N_r(x,y)} f_r(x) \ast g(y) \, dx \, dy \quad \text{Eq. 7}
\]

Integration in the joint probability space resolves problems related to the modeling of both strength and load distributions, and allows us to calculate the damage increment of each joint probability matrix cell. By performing the numerical integration over the entire probability range of zero to one in two parameters of load and strength, as expressed in Eq. 7, the total damage is calculated.

The 3-dimentional view of the joint probability density (JPD) for the same MR Rotating Swashplate is shown in Figure 10 with isolines of calculated Washplate, where the x-axis is the strength cumulative probability density (CPD), and the y-axis is the load CPD, while the z-axis represents the calculated JPD.

The 3-dimentional view of the calculated fatigue damage distribution is shown in Figure 11 with damage isolines, where the z-axis represents the calculated damage increment corresponding to the JPD of the cell.

As mentioned before, the joint probability density and the corresponding fatigue damage distribution can be calculated in the real dimensional space of the allowable cycles-load, even though this may not be exact. They are shown in Figures 12 and 13, and, as expected, their shapes are much more complex than those of the normalized joint probability space in Figures 10 and 11. These plots are very informative. The humps of JPD’s and damages from different regimes can be seen in the plots. Also, it can be noticed that the JPD on the high cycle region is high in Figure 12, but that the actual damage shown in Figure 13 in that region quickly tapers down because the number of allowable cycles is exponentially increasing. It also shows that the life reduction expected on the upper left side of the low cycle region is small.

**COMPOSITE WORST CASE USAGE SPECTRA**

Once the probability model for the load and strength is in place, CRT’s can be calculated by adding the flight usage data, the 3rd element of the fatigue life, as mentioned earlier.

In a typical Legacy method, a block spectrum representing the design operational usage of the aircraft is generated and used in the component fatigue life assessment by assuming that this block of flight usage is repeated during the entire lifetime of an aircraft.

A CWC spectrum is composed of occurrences and % time of flight maneuver regimes, typically in 100 flight hour block form. The histogram in Figure 14 shows the distribution of regime occurrences and % times of a CWC spectrum. Note that the CWC maneuver list contains the highest rate of occurrence for each regime that can occur in service for any mission or usage scenario anticipated for the designed helicopter, and that this results in a mission spectrum that cannot actually be flown and may be overly conservative [12].

This block spectrum approach makes regime usages inter-dependent of each other, i.e. all regimes are bound by their occurrences and % times, and hence make it difficult to accurately quantify the impact of the usage spectrum on the reliability of component life.
Figure 10. 3-Dimensional View of an Example Joint Probability Density in the Probability Space

Figure 11. 3-Dimensional View of an Example Fatigue Damage Distribution in the Probability Space

Figure 12. 3-Dimensional View of an Example Joint Probability Density in the Allowable-Load space
Figure 13. 3-Dimensional View of an Example Fatigue Damage Distribution in the Allowable-Load space

Figure 14. Example CWC Spectrum with Occurrences and % times of Regimes
Also, it is worth mentioning that the assessed life using this CWC spectrum is valid only when the predefined operational usage profile is maintained throughout the life of the component.

The same CWC usage spectra that had been used in the Legacy method were also used to calculate fatigue damages of selected components using the proposed joint probability fatigue damage model described above to compare and validate this portion of the methodology. This set of damages constitute the baseline of reliability based CRT before introducing HUMS data.

It is presumed that HUMS will provide accurate aircraft usage data, which will replace the CWC usage, and this HUMS part is discussed in a separate paper.

**DETERMINATION OF RELIABILITY LEVEL**

Once the probability fatigue damage model is developed and the CWC usage scenario is established, component fatigue damage at each joint probability matrix cell of the loads and strength can be calculated using the formula in Eq. 7. The distribution of joint probabilities and corresponding fatigue damages are shown in Figures 10 and 11 above for the MR Rotating Swashplate.

The reliability for the total component damage in Eq. 7 will be 1.0, or zero life, considering that the integration is carried out from zero to 1.0 cumulative probability in both the regime loads and material strength. From this, we can determine the component fatigue damage, i.e., the component retirement time, which meets the required reliability level of six-nines.

Considering that the information of damage, i.e., life, and corresponding joint probability of this damage are available for each joint matrix cell, the reliability level for a certain damage level can be obtained from the Damage-Joint Probability Vector (D-JP Vector). To do this, it is required to construct a damage-joint probability pairs from the joint probability matrix, and properly sort them to find the damage or life, corresponding to the required reliability level [6].

This process is somewhat difficult and cumbersome when repeated analyses are required for a trend analysis or for multiple components or usage cases due to the involvement of several manual steps such as sorting, searching required reliability levels and interpolations, etc. Also, this vector approach does not provide information for the source of changes in reliability levels.

An alternate approach has been investigated using the commonly used clipping method of the probability density curve to control the reliability level in the analysis. Here, a target reliability level is predetermined, "a priori", by one-side tail-clipping of pdf's where the two convoluting probability curves are engaged, i.e., the high-side of the loads and the low-side of the strength as shown in Figure 15. In the figure, pdf curves of 10 selected regimes and one strength curve in bold line at 3.0 million allowable cycles are shown with an exaggerated tail clipping of the 95% reliability level on the engaged sides.

Eq. 8 shows the revised formula with the engaged side tail clipping in the integration boundaries:

\[
D = \sum_r \int_0^{1-\varepsilon_l} \int_{\varepsilon_r}^1 n_r N_r(x, y) \tilde{f}_l(x) \tilde{g}(y) \, dx \, dy \quad \text{Eq. 8}
\]

where \( \varepsilon_l \) and \( \varepsilon_s \) are the tail-clipping of loads and strength, respectively.

It can be expected that this one-side tail clipping method, called as "a priori" method, will result in a more conservative damage because of the clipping of engaged sides only.

Reliability trends of number-of-nines vs. component life, for three typical dynamic components are shown in Figures 16, 17 and 18. As can be seen in these figures, the reliability level determined by the "a priori" method is slightly more conservative compared to the vector method in low-life regions, which are of our interest. Note that these reliability trend curves were obtained after the incorporation of confidence level using the Student T-Test method to take into account the effect of sample size. Details of this T-Test are described in the next section.
This "a priori" method enables control of the source of reliability changes, and is a much simpler and automated process for multiple runs, and has been used in the subsequent studies below.

Table 1 shows the calculated CRT’s for the three dynamic components using the “a priori” method, compared against the existing Legacy values.

Table 1. CRT Comparison between Legacy Method and Reliability Method for CWC Spectrum

Note that the reliability for the reliability based CRT’s from the strength and loads were five-nines (resulted from the multiplication of two 0.999995’s), and "one-nine credit" was added to make the final reliability to be six-nines due to the conservative CWC usage (without any statistical analysis). In general, the reliability based CRT’s are much greater than those of the Legacy values. Further modeling effort to incorporate the confidence level is described below.

**EFFECT OF SAMPLE DATA POINTS**

For any stochastic analyses using probabilities, it is important to have a sufficient number of input data points to achieve a high level of confidence because the end results are greatly affected by the available sample size. Also, as pointed out in the conclusion of the Round Robin 1 above, “a probability analysis will result in the expected value, i.e. the 50% confidence level of reliability analysis by definition, if no special attempt is incorporated” [6].

Considering that the realities of most of the rotorcraft component fatigue tests rely on a few test data points, the confidence on the fatigue strength is of particular concern. Figure 19 shows an example of a typical S-N curve for a rotorcraft dynamic component.

Without including a provision of confidence level in the strength modeling, it may be difficult to attain the required high level of reliability due to the uncertainty from the limited number of fatigue test data points. The current Army design standard requires a 95% confidence level in the fatigue analysis [2].

In the proposed methodology, it was attempted to incorporate the confidence level in the strength probability modeling to alleviate the uncertainty due to the limited number of test data points using the Student T-Test method.
Margin of errors of a component mean strength can be calculated using the Student T-Test as shown in Eq. 9, and the mean strength was reduced by the amount of error margin corresponding to the number of test data points in the study [2].

\[ \hat{x}_n - \frac{\sigma_n}{\sqrt{n}} t_{\alpha/2,v} \leq \mu \leq \hat{x}_n + \frac{\sigma_n}{\sqrt{n}} t_{\alpha/2,v} \]  
Eq. 9

The mean strength (\(\mu\)) and the standard deviation (\(\sigma\)) are the two primary parameters defining the typical S-N curve formula shown in Eq. 5, and even though the standard deviation will have its own error margins as well, the error margins of only the mean strength were considered in this first attempt of including confidence levels in the model. Incorporation of error margins for the standard deviation is a subject of future study.

Considering that the existing Legacy method did not have any provision for confidence levels and most of existing fatigue tests have only a few data points results will be too drastic change of assessed fatigue life at the 95% two-side confidence level as stated in Reference [2]. Therefore, a somewhat lower confidence level of one-side 90% confidence level was used in this V&V study for development of a probabilistic fatigue damage model. This confidence level needs a further review in the future by all parties to establish an industry wide standard.

Figure 20 shows a typical downward trend of the calculated t values and corresponding margin of errors when the available number of data point are increasing.

\[ x_n - \frac{\sigma_n}{\sqrt{n}} \left[ \alpha_{1,2,v} \right] \leq x_n \leq x_n + \frac{\sigma_n}{\sqrt{n}} \left[ \alpha_{1,2,v} \right] \]  
Eq. 9

It can be observed from these plots that the incorporation of a confidence level by the Student T-Test substantially reduced the final CRT’s for the three components as expected. Still, in general, reliability based CRT’s are greater when compared to legacy CRT’s. However, in the Pitch Control Horn case, the large COV (0.159) made its reliability based CRT smaller than the legacy value (11,000 hours).

A rapidly increasing trend of the CRT when the number of specimens is increasing provides a strong incentive for more fatigue tests.
After the Strength adjustment, reduced after the Strength adjustment. It can be seen from the table and plot that the CRT’s have been substantially reduced after the Strength adjustment.

Table 2 and Figure 24 show the summary of CRT comparisons between the Legacy method and the reliability based method before and after the strength adjustment based on the number of fatigue test specimens discussed above. It can be seen from the table and plot that the CRT’s have been substantially reduced after the Strength adjustment.

Table 2. CRT Comparison between Legacy Method and Reliability Method for CWC Spectrum before and after the Strength Adjustment

<table>
<thead>
<tr>
<th>Reliability (Number of 9’s)</th>
<th>Legacy</th>
<th>Reliability Based before</th>
<th>Reliability Based after</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Strength</td>
<td>3</td>
<td>5.3</td>
<td>5.3</td>
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<tr>
<td>Loads</td>
<td>2</td>
<td>5.3</td>
<td>5.3</td>
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<tr>
<td>Usage (CWC)</td>
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<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fatigue Life (Hours)</th>
<th>MR Rotating Swashplate</th>
<th>MR Pitch Control Horn</th>
<th>MR Shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legacy</td>
<td>10,011</td>
<td>11,027</td>
<td>25,000</td>
</tr>
<tr>
<td>Reliability Based (before t-test adjustment)</td>
<td>50,319</td>
<td>28,636</td>
<td>179,298</td>
</tr>
<tr>
<td>Reliability Based (after t-test adjustment)</td>
<td>93,449</td>
<td>9,369</td>
<td>40,594</td>
</tr>
</tbody>
</table>

RELIABILITY TREND ANALYSIS

With the revised probability model of the fatigue strength using the Student T-Test method to incorporate the confidence level, we can analyze reliability trends and compare the final reliability based CRT’s against the Legacy CRT’s by using the CWC usage spectrum.

The same three dynamic components, MR Rotating Swashplate, Pitch Control Horn and MR Shaft, for a mid-size utility helicopter model were selected to examine the proposed reliability modeling by comparing CRT’s and quantified reliabilities with those of the existing Legacy values based on the following conditions:

- Reliability based CRT’s are assessed using the joint probability model of flight loads and material strength.
- Effect of number of specimens are incorporated in the material strength modeling using the 90% one-side confidence of margin of error defined by the Student T-Test formula on the mean strength only.
- Other than the reliability trending analysis, the reliability level was fixed to the six-nines in the study using the “a priori” method.

Note again that, because the conservative CWC usage had been considered as adding one-nine reliability of the component life before [3], “one-nine credit” was added to the actual reliabilities quantified throughout the trend study. This added reliability will be removed when the CWC spectrum is replaced by the spectrum generated using HUMS data in the Part 2 study.

Figures 25 and 26 show the reliability trends of the selected three components when the number of...
reliability nines changed. As can be seen in the figures, fatigue lives do not change much when the number of nines is greater than 4, while the reliability level drops sharply with a small change in lives in this region.

Pitch Control Horn compare to the Swashplate as explained earlier. The deterministic model in the Legacy method could not accurately address these differences.

The reliability trend shown in Figure 26 indicates a need for more detail study to explain the big difference between the reliability based CRT and the Legacy CRT of the MR Shaft. This substantial difference came from the limitation of Legacy method in accurately calculating the damage when the regime load is close to the working endurance limit as explained later when we investigate individual regimes.

It was demonstrated that the flight loads and the material strength provide two-nines and three-nines reliability for the Legacy method as indicated earlier. Based on this assumption, a method was proposed to further reduce the mean strength from the usual 3-σ to the level of four-nines of reliability to meet the required six-nine reliability when HUMS usage data was to be used [10]. In this sense, it is also worthwhile to investigate how the varying reliability levels of flight loads and fatigue strength influence the reliability of the assessed component life.

The trends of CRT’s for various combinations of reliability levels of the loads and strength were studied to understand their impact on the component life. Three typical combinations were considered: 1) the two reliability levels were identical while varied equally, 2) the strength reliability level was kept constant at 0.9999995 while the other reliability level of loads varied, and finally 3) the loads reliability level was kept constant at 0.9999995 while the strength reliability level varied.

As can be seen in the figures above, the symbols represent the Legacy CRT’s. Note that the Legacy CRT’s of MR Rotating Swashplate and Pitch Control Horn are fairly close in Figure 25, while there is a substantial difference between their reliability based CRT’s. This is due to the much flatter S-N curve of the Pitch Control Horn shown in Figure 19 compared to the S-N curve of the MR Rotating Swashplate in Figure 31, as well as the relatively higher COV value from having more scatter of the fatigue test data in the

Figure 25. Reliability Based Component Fatigue Life vs. Reliability Level – MR Swashplate and Pitch Control Horn

Figure 26. Reliability Based Component Fatigue Life vs. Reliability Level – MR Shaft

In the figures above, the symbols represent the Legacy CRT’s. Note that the Legacy CRT’s of MR Rotating Swashplate and Pitch Control Horn are fairly close in Figure 25, while there is a substantial difference between their reliability based CRT’s. This is due to the much flatter S-N curve of the Pitch Control Horn shown in Figure 19 compared to the S-N curve of the MR Rotating Swashplate in Figure 31, as well as the relatively higher COV value from having more scatter of the fatigue test data in the

Figure 27. Reliability Trends for Various Combinations of Reliabilities in Loads and Strength – MR Swashplate
DAMAGE OF INDIVIDUAL REGIMES

As indicated by the formula in Eq. 8, the total damage is the summation of damages incurred by individual regime loads. These individual regime damages for the three components are compared between the Legacy method and the reliability based method in Figures 30, 33 and 34.

As expected, a few dominant regimes produce most of the damage in both methods, and in general, the reliability based regime damages are much smaller compared to the counter parts of the Legacy method.

Note that there are several regimes showing substantial differences between the two methods. The following regimes are such cases for the MR Rotating Swashplate in Figure 30:

- H-Speed L Flts (19-21),
- Regime ID 22 (PP Descent),
- Regime ID 30 (Approach).

All of these regimes have their TOS loads in the Legacy method close to the reduced endurance limit. In the Legacy method, when the TOS regime loads are close to the endurance limit, the Legacy method could miss potentially damaging regimes entirely, or overestimate damages, “all-or-no” damages, for these border-line regimes.

Figure 31 shows the working S-N curve of the MR Rotating Swashplate where the reduced endurance limit is 1716 lbs. In Figure 32, pdf's of some of the borderline regimes are shown together with the three pdf's of the strength at selected allowable cycles to illustrate their lightly overlapped joint probability situations. All of these regimes have their TOS loads used in the Legacy method around the endurance limit between 1500 to 2000 lbs. Depending on the situation as to whether the load is greater than or less than the endurance limit, a “full credited (all)” damage or zero damage, respectively, i.e. “all-or-no” damage, will result in the Legacy method, while the reliability based method will assess the damage based on its joint probability. Further details are explained below. Figures 35 and 36 illustrate the similar situations for the Pitch Control Horn and the MR Shaft, respectively.

Figure 37 shows an example of “no” damage case where the TOS load level of the 30° L. Turn (34) maneuver regime on the Pitch Control Horn is slightly smaller than its endurance limit, resulting in “no” damage in the Legacy method, while the substantial joint probability from the overlapping of the two PDF’s shown in the figure produces a substantial regime damage in the reliability based method (see Figure 33).
Figure 30. Comparison of Regime Damages between Legacy and Reliability Methods – MR Rotating Swashplate

Figure 31. Working S-N Curve – MR Rotating Swashplate

Figure 32. PDF’s of Borderline Regime Loads with Strength PDF’s at Three Allowable Cycles – MR Rotating Swashplate
Figure 33. Comparison of Regime Damages between Legacy and Reliability Methods – Pitch Control Horn

Figure 34. Comparison of Regime Damages between Legacy and Reliability Methods – MR Shaft
from the overlapping of the two PDF’s shown in the figure produces a negligible amount of regime damage in the reliability based method as shown in Figure 34.

**SUMMARY AND CONCLUSIONS**

The various approaches and subsequent analysis results of the proposed probability modeling of flight loads and fatigue strength for the reliability based fatigue damage assessment are described and discussed.

A probability model of component fatigue damage was developed based on the joint probability of fatigue strength and flight loads. The model was revised based on the error margin of mean strength to incorporate the confidence level of the fatigue strength. Followings are highlights of the methodology:

- The joint probability and corresponding fatigue damage of each joint probability matrix cell are calculated for each regime.
- The fatigue damage for each regime is computed by a numerical integration in the joint probability space.
- The required reliability level is controlled “a priori” (pre-determined) by one-side tail-clipping the engaged pdf sides of loads and strength.
- The mean strength is reduced based on its error margins due to the limited number of fatigue test specimens, and to incorporate the confidence level in the process.

The fatigue lives and their reliability levels with the CWC usage spectrum for the selected three dynamic components: the MR Rotating Swashplate, the Pitch Control Horn and the MR Shaft were assessed and compared to the Legacy method.

Reliability based fatigue lives were generally greater than the current Legacy lives, while the reliability
based life of the Pitch Control Horn was smaller due to the high scatter in fatigue strength of the component.

Through fairly extensive studies, it was demonstrated that the proposed method provides more accurate reliability trends and reference CRT values due to the reliability based methodology.

RELIABILITY BASED COMPONENT LIFE ASSESSMENT USING HUMS – PART 2 STUDY

The probability model of component fatigue damage described in this paper was further used to evaluate and address the issues in using HUMS for the reliability based fatigue life assessment. Considering the volume of the work and difference in its content, this second part of the study results will be presented in a separate paper, but a few analysis results from this part 2 study are summarized below:

- Using the recorded HUMS data of four aircraft, new and updated usage spectra for individual aircraft were developed to replace the CWC spectrum used in the part 1 study to assess CRT’s. Based on these new CRT’s, the two sources of CRT changes, one from the probability modeling and the other from HUMS were identified and compared.

- HUMS data of derivative model aircraft from a larger fleet size were used to assess CRT’s, and the CRT trend with respect to the accumulated flight hours were analyzed.

- The CRT data of individual aircraft from this larger pool of HUMS data were used to generate a pdf to develop a representative CRT which could be used to adjust the fleet-wide component life.

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